Poisson Distribution

Consider a random variable that gives the number of occurrences over a specified time or space. The specified time may be a 1-day period or a 1-week period and the specified space may be a 1 km radius. This random variable X is a random variable with Poisson distribution denoted by $X \sim Po(\lambda)$, where λ is the average number of occurrences in the specified time or space.

Here are the properties of a Poisson experiment that focus on occurrences in a given interval of time or a region of space:

1. the probabilities of occurrences for any two intervals of the same length are equal; and
2. the occurrence or non-occurrence in a given interval does not in any way affect the occurrence or non-occurrence in any other interval.

[redacted]

For a discrete random variable X that follows a Poisson distribution denoted by $X \sim Po(\lambda)$, the probability mass function is

$$P(x) = \begin{cases} \dfrac{e^{-\lambda} \cdot \lambda^{x}}{x!}, & x = 0, 1, 2, 3, \ldots \\ 0, & \text{otherwise} \end{cases}$$

where λ is the average number of occurrences in a given interval of time or a region of space and e is Euler's number, approximately equal to 2.71828.

The mean and the variance of a Poisson random variable are both equal to λ. That is, for a Poisson distribution,

$$E[X] = \lambda \qquad \text{and} \qquad Var[X] = \lambda.$$

[redacted]

Example 2.8

Suppose the number of customers served by a bank teller follows a Poisson distribution and the average number of customers served by a bank teller in a 15-minute period is 4.

1. Find the probability mass function for the number of customers served by this bank teller in a 15-minute period.
2. Find the probability of her serving exactly 3 customers in a 15-minute period.
3. Find the probability of her serving at least 2 customers in a 15-minute period.

Solution.

In this case, the number of customers served is a Poisson random variable denoted by $X \sim Po(\lambda)$.

1. The probability mass function is given by

$$P(X = x) = \begin{cases} \dfrac{e^{-4} \cdot 4^{x}}{x!}, & x = 0, 1, 2, 3, \ldots \\ 0 & , \text{otherwise.} \end{cases}$$

2. $P(X = 3) = \dfrac{e^{-4} \cdot 4^{3}}{3!} = .1954$

3. $P(X \geq 2) = P(X = 2) + P(X = 3) + \ldots$

However, note that $P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)]$.

Thus, $P(X \geq 2) = 1 - \left(\dfrac{e^{4} 4^{0}}{0!} + \dfrac{e^{-4} 4^{1}}{1!} \right) = 1 - (.0183 + .0733) = .9083$

[redacted]